LETTER

Decodability of Network Coding with Time-Varying Delay and No Buffering

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SUMMARY We show the sufficient conditions for coding by nodes such that every sink can decode all information multicasted by the single source when there exists time-varying delay of information transmission at links in the network coding.
key words: *time-varying delay, network coding, linear coding, multicast*

1. Introduction

In the traditional information transmission through a network, an intermediate node was only allowed to duplicate input information and output the duplication. In contrast, in the transmission with the network coding, an intermediate node is able to code the input information from multiple links and outputs the coded information. The multicast using the network coding can achieve higher information rate than the traditional multicast [1]. Hence the network coding is attracting much attention as a method for multicast. Furthermore, it is known that linear coding in each node is sufficient to multicast in such a rate [2]. Accordingly in this paper, we only deal with the linear network coding. In the following, the network coding refers to the linear network coding, and the single source multicast is considered.

For practical realization of the network coding, we should consider delay in the information transmission and delay is not always constant in time. Because of time-varying delay, a sink may sometimes fail to decode transmitted information depending on the structure of the network and coding. However, in most of past research of the network coding, it was assumed that there is constant delay that does not change in time or that there is no delay of information transmission [1]–[4]. The drawback in these researches is that each node have to wait for coding until all the information reaches to the node when there is time-varying delay. This means that the buffer for coding by nodes is necessary and that transmission delay unnecessary accumulates in every node.

As a research considering time-varying delay, Chou et al. proposed a scheme by employing *generation* [5]. In the scheme, packets containing information multicasted at the same time are said to be in the same generation. Each packet header includes the index of generation and coefficients of linear combination. For the packet arriving at each intermediate node within a fixed time interval, the coding is done among packets belonging to the same generation. We call their scheme the generation by generation network coding. However, in their scheme, because the coding of each generation is only permitted, it is necessary to transmit as many packets as the number of generations accumulated in each node. Hence the bandwidth is wasted. Also, the buffer to synchronize the same generation packets is necessary.

We can consider a scheme that allows coding among different generations in order to decrease the amount of buffers and to use the bandwidth efficiently compared with the generation by generation network coding. Recently, Halloush et al. proposed a approach called *multi-generation mixing* (MGM) that allows coding among generations [6], [7]. However, they did not clarify the condition under which every sink can decode all the information. They only verified the decodability by computer simulations in limited scenarios.

We propose the scheme in which information multicasted at the different time is coded by intermediate nodes without buffers when there is time-varying delay. Because coding is permitted among all generations, when the same amount of information as the generation by generation network coding [5] is multicasted, the amount of transmitted packets decreases, that is, the bandwidth is more efficiently used. In our scheme, it is important to clarify the condition under which every sink can decode all the information and the condition under which a sink fails to decode. We show the example when all information cannot be decoded, and sufficient conditions for coding by nodes such that every sink can decode all information multicasted by the single source.

The rest of paper is organized as follows. In Sect. 2 we explain the definitions and model of a network with time-varying delay. In Sect. 3, we introduce an example in which information cannot be decoded by a sink due to the time-varying delay. After that we show sufficient conditions for every sink to decode all information, and analyze decoding delay in Sect. 3. Section 4 gives conclusion and future tasks.

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2. Preliminaries

2.1 Basic Notation

We shall assume a capacity of link that connects nodes together as a unit capacity, that is, it can only transmit one of the elements in a finite field $\mathbf{F}_q$ per unit time. A directed acyclic network that permits multiedge is represented by $G = (V, E)$ in this paper, where V and E represent the sets of nodes and links, respectively. We also represent the only source by $s \in V$, the dummy source by $s' \in V$ and the set of sinks by $T = \{t_1, t_2, \cdots, t_d\} \subset V$. It is well known that when the minimum of max-flow between the source s and each sink $t_i \in T$ is denoted by h', the maximum amount of a symbol that can be multicasted from the source s to all sinks is h' in the network where all link capacities are 1 [1].

In this paper, we assume the number of input links from the dummy source s' to the source s to be $h \le h'$, and the dummy source s' sends h symbols in $\mathbf{F}_q$ to the source s without coding per unit time. Also, we index h symbols from one to h, and the source s sends them after coding. The set $\gamma^-(v)$ denotes the set of input links to a node $v \in V$, and start(e) denotes the node whose output link is e. Then the map m_e associated to a link e of G is defined as

$$m_e : \gamma^-(\text{start}(e)) \to \mathbf{F}_q.$$

The m_e is called as the *local coding vector* (LCV), it decides the coefficient of a linear relation between inputs and outputs of each node [8].

2.2 Network Coding without Delay

In Sect. 2.2, we shall consider the case that there is no delay in information transmission. The information that we want to multicast is represented by an *information vector* $\vec{i} \in \mathbf{F}_q^h$. The j-th row of $\vec{i}$ denotes the j-th symbol from the dummy source s'. Let $y(e)$ be the symbol that flows through the output link e of an intermediate node (except for the dummy source s') at a certain time, and we have

$$y(e) = \sum_{e_i \in \gamma^-(\text{start}(e))} m_e(e_i) y(e_i). \tag{1}$$

Also, for all $\vec{i}$ and $y(e)$ for a link e, a column vector $\vec{b}(e) \in \mathbf{F}_q^h$ which satisfies

$$y(e) = \vec{i} \cdot \vec{b}(e) \tag{2}$$

is defined as the *global coding vector* (GCV). Note that the GCV is uniquely determined, because a GCV represents a linear map sending a vector in $\mathbf{F}_q^h$ to a symbol in $\mathbf{F}_q$ [8]. The GCV $\vec{b}(e)$ represents the influence of coding by all upstream nodes of the link e. For links e_i $(i = 1, \ldots, h)$ from the dummy source s' to the source s,

$$\vec{b}(e_i) = [0^{i-1}, 1, 0^{h-i}]^T \tag{3}$$

holds, and for the rest of links

$$\vec{b}(e) = \sum_{e_i \in \gamma^-(\text{start}(e))} m_e(e_i) \vec{b}(e_i) \tag{4}$$

holds [8]. In the following, the GCV is used for referring to coding when there is no delay.

2.3 Network Coding with Time-Varying Delay

In Sect. 2.3, we shall consider that there is time-varying delay in information transmission. We shall use discrete time indexing. The delay in information transmission between nodes is assumed to be non-negative integer multiple of unit time. In addition, an input symbol to a link is always output before its succeeding input symbols and there is no loss or error of symbols on a link. Also, at most one symbol is output from a link per unit time. The dummy source s' sends h symbols per unit time to the source s from time 1 to ℓ, but does not send symbols after time ℓ. We assume that there is no delay in links between s' and s. The total number of $\mathbf{F}_q$ symbols transmitted from s' is $h\ell$. We shall represent these $h\ell$ symbols by the following $h \times \ell$ matrix denoted by the *information matrix* $\mathbf{i}$. The (j, k) component of $\mathbf{i}$ represents the j-th symbol among h symbols transmitted from s' at the time k.

Let $y_n(e)$ be the output symbol from a link e at time n. If the link e does not output a symbol at time n because of time-varying delay, then we define $y_n(e) = 0$. We use the same LCV m_e at every time slot. If a node does not receive a symbol from an input link, then the node assumes that the link outputs 0. Under these assumptions, we have following relation

$$y_n(e) = \sum_{e_i \in \gamma^-(\text{start}(e))} m_e(e_i) y_m(e_i). \tag{5}$$

Equation (5) implies that input to link e at time m is output at time n. Equation (5) corresponds Eq. (1), which assumes no delay in information transmission.

For all $\mathbf{i}$ and $y_n(e)$ on a link e at time n, a matrix $b_n(e)$ which satisfies

$$y_n(e) = \mathbf{i} \cdot b_n(e) \tag{6}$$

is defined as the *global coding matrix* (GCM), where the inner product of matrices A and B is defined as $\text{Tr}[AB^T]$. Equation (6) corresponds to Eq. (2). The GCM is uniquely determined as well as the GCV, because a GCM represents a linear map sending a matrix in $\mathbf{F}_q^{h \times \ell}$ to a symbol in $\mathbf{F}_q$. Furthermore, in the case there is no output from a link e at time n, $b_n(e)$ becomes the zero matrix. Since symbols are sent from time 1 to ℓ, the GCM is an $h \times \ell$ matrix, the same as the information matrix $\mathbf{i}$. In addition,

$$b_n(e) = \sum_{e_i \in \gamma^-(\text{start}(e))} m_e(e_i) b_m(e_i) \tag{7}$$

holds corresponding to Eq. (4). For $b_n(e_i)$ of a link e_i $(i = 1, \ldots, h)$ from the dummy source s' to the source s, the n-th column is $[0^{i-1}, 1, 0^{h-i}]^T$, and the other components are 0.

We include GCM's in packet headers for enabling every sink to decode **i**. A sink can recognize how linearly combined symbol is included in the arrived packet. If ℓ, which is the time of last symbol transmitted from the dummy source, is large, then the packet header containing the GCM would be bigger, but we explain a method to avoid this problem in Sect. 3.2.

3. Sufficient Conditions for Every Sink to Decode All the Transmitted Symbols

In Sect. 3, Sect. 3.1 gives an example of a network where the symbol cannot be decoded by a sink due to time-varying delay. For a network where the source receives two symbols per unit time and multicasts them, Sect. 3.2 gives a sufficient condition that all transmitted information can be decoded regardless of time-varying delay. In Sect. 3.3 we show that when the source multicasts three or more symbols, the sufficient condition in Sect. 3.2 is insufficient. Section 3.4 gives sufficient conditions that all transmitted information can be decoded in the network where the source receives more than two symbols per unit time and multicasts them.

3.1 Example of a Network Where Symbols Cannot be Decoded by the Sink Due to the Time-Varying Delay

Suppose that we have LCV's on a network that allows every sink to decode the transmitted information when there is no delay. We shall show an example of time-varying delay preventing a sink from successful decoding with such LCV's. Such example requires us to find sufficient conditions for successful decoding with every possible time-varying delay. Figure 1 is an example of a network where the symbol cannot be decoded by the sink due to the time-varying delay. In Fig. 1, h is 1 and the characteristic of the finite field is five or more. The source receives only symbol $a_i \in \mathbf{F}_q$ at time i. The source sends a_i at time i for $i = 1, 2, 3$. The links $e_i\,(i = 1, \cdots, 5)$ and the nodes $s, 1, 2$ and t are defined as Fig. 1. The LCV's are defined as follows: $m_{e_2}(e_5) = 1, m_{e_3}(e_5) = 1, m_{e_4}(e_5) = -1, m_{e_1}(e_2) = 1, m_{e_1}(e_3) = 1, m_{e_1}(e_4) = 1$. Assume that we have time-varying delay and output symbols from each link as shown in Table 1. The symbols received by the sink are output from e_1. Observe that a_1 and a_2 are not decodable. Such a situation can occur in the random network coding [9].

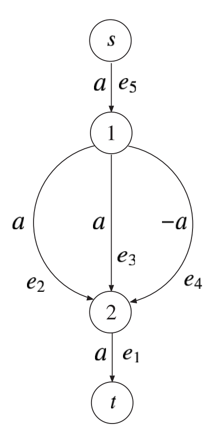

Fig. 1 Network where transmitted symbol cannot be decoded.

3.2 Sufficient Condition for Two Symbols

We shall present a sufficient condition to decode all transmitted information when the source s receives two symbols per unit time, multicasts them and there is time-varying delay in a network. The set $E_i\,(i = 1, \ldots, h)$ of links is defined as $E_i = \{e \in E \mid \text{the } i\text{-th component of } \vec{b}(e) \text{ is non-zero}\}$. We introduce the sufficient condition.

Condition 3.1: For all $i = 1, \ldots, h$ and every link e except the links from s' to s, the cardinality of the set $\{e' \in E_i \mid e'$ is an immediate upstream of the link e and $m_e(e')$ is non-zero$\}$ is at most 1.

We remark that the butterfly network [1] satisfies Condition 3.1.

Lemma 3.1: We fix an input link e to a sink in this lemma. For the network which satisfies Condition 3.1, the following claims hold.

(1) If the i-th row of the GCV $\vec{b}(e)$ is 0, then the i-th row of GCM $b_n(e)$ is 0 for all n.
(2) If the i-th row of the GCV $\vec{b}(e)$ is non-zero value x, there exists unique n for each j such that the (i, j) component of the GCM $\vec{b}_n(e)$ is non-zero, which is denoted by $t_i(j)$. The (i, j) component of such $b_n(e)$ is x.
(3) The function $t_i(j)$ is strictly increasing.

Proof: (1) Assume that there are two or more links in an immediate upstream of a link f and they belong to E_i. If the symbols that flow through each link in E_i are mutually cancelled by linear combination on start(f) when there is no delay, then the i-th row of $\vec{b}(f)$ is 0. However, when there is time-varying delay, the i-th row of $b_n(f)$ may be non-zero. By Condition 3.1 there does not exist such a situation. There are no other cases in which the i-th row of $\vec{b}(f)$ is zero and that of $b_n(f)$ is non-zero. Therefore we obtain (1) of Lemma 3.1.

(2) If Condition 3.1 is satisfied, then there is only at most one path from the source s ending at a link e such that every link f on the path has a non-zero i-th component of $\vec{b}(f)$. This means that the i-th symbol from s' at time j does not split into two or more time slots nor disappear. Hence we obtain (2) of Lemma 3.1.

(3) In addition to (2), because an input symbol to a

Table 1 Output symbols from each link outputs in Fig. 1.

time	e_2	e_3	e_4	e_1
1	0	a_1	$-a_1$	0
2	a_1	a_2	0	$a_1 + a_2$
3	a_2	a_3	$-a_2$	a_3
4	a_3	0	$-a_3$	0

link is always output before its succeeding input symbol and there is no loss or error of symbols on a link, we obtain (3) of Lemma 3.1. ■

According to Lemma 3.1, a GCM can be compactly recorded at a packet header by using two kinds of values. These values are the value of non-zero component in each row of the GCM, and the column of the non-zero component, in other words, the time that the symbol is transmitted from the dummy source (if there are no non-zero components in the row, these values are zero). Thus $2h$ values are sufficient for recoding a GCM in a packet header.

Because the butterfly network satisfies Condition 3.1, every sink can be decoded all transmitted information even if there is time-varying delay by the following theorem.

We consider the following decoding procedure. We define a source symbol as a symbol flows from the dummy source s' to the source s, that is, an information symbol to be multicasted. Also we define an incoming symbol as a symbol flows on an input link to the sink. Consider an arbitrary fixed sink. Two input links to the sink whose GCV's are linearly independent are chosen. We pick the earliest incoming symbol from each link to the sink containing an undecoded source symbol. We decode the undecoded source symbols in the pair of those two incoming symbols.

Theorem 3.1: Assume that a sink can decode all the source symbols if there is no delay in a network with Condition 3.1 and that h is 2. Fix two links whose GCV's are linearly independent. Then even if there is time-varying delay, whenever the sink receive incoming symbols containing an undecoded source symbol from both links, the undecoded source symbol contained in them can be decoded.

Proof: Let e_1 and e_2 be input links to the sink such that $\vec{b}(e_1) = [x_1, y_1]^T$ and $\vec{b}(e_2) = [x_2, y_2]^T$ are linearly independent. We convert a GCM into a row vector such that the (n, m) component of the GCM is the $(\ell(n-1)+m)$-th component of the row vector. All the row vectors have size 2ℓ. In all the row vectors, the components from the first (the $(\ell+1)$-st) to the ℓ-th (the 2ℓ-th) correspond to the first (second) source symbol, respectively. Also, the i-th ($(i+\ell)$-th) component of the row vectors corresponds to the first (second) source symbol transmitted from the source at time i for $1 \leq i \leq \ell$, respectively. We also define the reduced GCM as the GCM in which the components corresponding to already decoded symbols are set to 0.

Consider the two row vectors corresponding to the non-zero reduced GCM's pair received from both links. It is assumed that p first source symbols and q second source symbols have been decoded by the time the sink decodes source symbols corresponding to the non-zero reduced GCM's pair. Accordingly the sink has decoded the source symbols corresponding to the components from the first to the p-th component and from the $(1+\ell)$-th to the $(q+\ell)$-th in the row vectors. Thus the those components in the row vectors are 0. Then by Lemma 3.1 the row vectors can only have one of the nine forms in Table 2. In any pattern, the number of the undecoded source symbol is 1 in every pair of incoming symbols or the two vectors are linearly independent because of the linear independence for the GCV's. Accordingly the undecoded source symbols can be decoded.

Therefore every time the sink received an incoming symbol pair containing the undecoded source symbol from both links, the source symbol contained in them can be decoded. ■

Table 2 The row vectors from two links.

pattern	row vector from e_1	row vector from e_2
1	$(0^p, x_1, 0^{\ell-1-p+q}, y_1, 0^{\ell-1-q})$	$(0^p, x_2, 0^{\ell-1-p+q}, y_2, 0^{\ell-1-q})$
2	$(0^p, x_1, 0^{\ell-1-p+q}, y_1, 0^{\ell-1-q})$	$(0^p, x_2, 0^{2\ell-1-p})$
3	$(0^p, x_1, 0^{\ell-1-p+q}, y_1, 0^{\ell-1-q})$	$(0^{\ell+q}, y_2, 0^{\ell-1-q})$
4	$(0^p, x_1, 0^{2\ell-1-p})$	$(0^p, x_2, 0^{\ell-1-p+q}, y_2, 0^{\ell-1-q})$
5	$(0^p, x_1, 0^{2\ell-1-p})$	$(0^p, x_2, 0^{2\ell-1-p})$
6	$(0^p, x_1, 0^{2\ell-1-p})$	$(0^{\ell+q}, y_2, 0^{\ell-1-q})$
7	$(0^{\ell+q}, y_1, 0^{\ell-1-q})$	$(0^p, x_2, 0^{\ell-1-p+q}, y_2, 0^{\ell-1-q})$
8	$(0^{\ell+q}, y_1, 0^{\ell-1-q})$	$(0^p, x_2, 0^{2\ell-1-p})$
9	$(0^{\ell+q}, y_1, 0^{\ell-1-q})$	$(0^{\ell+q}, y_2, 0^{\ell-1-q})$

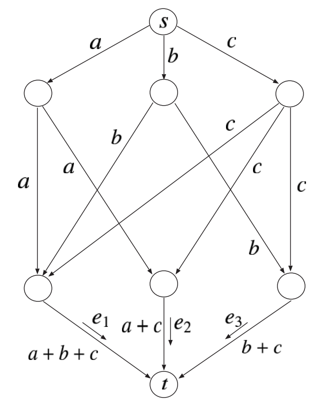

Fig. 2 Example of a network where symbols cannot be decoded though Condition 3.1 is satisfied.

Corollary 3.1: By the decoding procedure and Theorem 3.1, the sink can decode all the source symbols when the source multicasts two symbols per unit time. ■

3.3 Insufficiency of Condition 3.1 for Three Symbols

When the source receives more than two symbols per unit time and multicasts them, even if Condition 3.1 is satisfied, there are networks where all symbols cannot always be decoded by all sinks. The example is shown in Fig. 2. In the network of Fig. 2, Condition 3.1 is satisfied. The source receives three symbols $a_i, b_i, c_i \in \mathbf{F}_2$ $(i = 1, 2)$ at time i, and multicasts them to the sink. The e_1, e_2 and e_3 denote input links to the sink as in Fig. 2. When delay is time-varying in this network, there are cases that transmitted symbols cannot be decoded. Table 3 shows output symbols from each link in such a case. Because the system of linear equations is underdetermined, not all symbols can be decoded.

3.4 Sufficient Conditions for More than Two Symbols

Condition 3.1 is insufficient for more than two symbols be-

Table 3 Output symbols from each link outputs in Fig. 2.

time	e_1	e_2	e_3
1	$a_1 + c_1$	$a_1 + c_1$	$b_1 + c_1$
2	$a_2 + b_1$	$a_2 + c_2$	$b_2 + c_2$
3	$b_2 + c_2$	0	0

cause of the previous counter example in Sect. 3.3. Accordingly, we shall present an additional condition that becomes a sufficient condition combined with Condition 3.1. When there is no delay, consider an arbitrary fixed sink. Let $e_1, e_2, \cdots, e_h$ be input links to the sink such that $\vec{b}(e_1), \vec{b}(e_2), \cdots, \vec{b}(e_h)$ are linearly independent. Define $S(\vec{v})$ as the set of indices whose components in the vector $\vec{v}$ are non-zero. We introduce Condition 3.2.

Condition 3.2: For $i = 1, \ldots, \lfloor h/2 \rfloor$ and input links e_j ($j = 1, \cdots, h$) to the sink, the following equation holds by renumbering of indices:

$$\left| \bigcup_{j=2i-1}^{2i} S(\vec{b}(e_j)) \setminus \bigcup_{j=1}^{2(i-1)} S(\vec{b}(e_j)) \right| = 2. \tag{8}$$

Also, if h is an odd number, the following equation holds in addition to Eq. (8):

$$\left| S(\vec{b}(e_h)) \setminus \bigcup_{j=1}^{h-1} S(\vec{b}(e_j)) \right| = 1. \tag{9}$$

Theorem 3.2: Assume that a sink can decode all the source symbols if there is no delay in a network with Conditions 3.1 and 3.2. Choose two links e_{2i} and e_{2i-1} ($1 \leq i \leq \lfloor h/2 \rfloor$) which satisfy Eq. (8). Assume that there is time-varying delay and the sink received an incoming symbol containing undecoded source symbols corresponding to $\bigcup_{j=2i-1}^{2i} S(\vec{b}(e_j)) \setminus \bigcup_{j=1}^{2(i-1)} S(\vec{b}(e_j))$. Then after source symbols corresponding to $\bigcup_{j=1}^{2(i-1)} S(\vec{b}(e_j))$ in the incoming symbol are decoded, the undecoded source symbols contained in them can be decoded. Similarly when h is odd and the sink received an incoming symbol containing undecoded source symbols corresponding to $S(\vec{b}(e_h)) \setminus \bigcup_{j=1}^{h-1} S(\vec{b}(e_j))$ from e_h, after a source symbols corresponding to $\bigcup_{j=1}^{h-1} S(\vec{b}(e_j))$ in the incoming symbol are decoded, the undecoded source symbol contained in it can be decoded.

Proof: If Condition 3.2 is satisfied, the number of the source symbol corresponding to $\bigcup_{j=2i-1}^{2i} S(\vec{b}(e_j)) \setminus \bigcup_{j=1}^{2(i-1)} S(\vec{b}(e_j))$ in incoming symbols from e_{2i} and e_{2i-1} is at most 2. Therefore the source symbols can be decoded by using the procedure in Sect. 3.2. On the other hand, when h is odd, the number of the source symbol corresponding to $S(\vec{b}(e_h)) \setminus \bigcup_{j=1}^{h-1} S(\vec{b}(e_j))$ in an incoming symbol from e_h is at most 1. Accordingly, after source symbols corresponding to $\bigcup_{j=1}^{h-1} S(\vec{b}(e_j))$ in the incoming symbol from e_h are decoded, the undecoded source symbols contained in them can be decoded. ■

Corollary 3.2: By the decoding procedure and Theorem 3.2, the sink can decode all the source symbols when the source multicasts three or more symbols. ■

4. Conclusion and Future Tasks

The past researches of network coding did not adequately consider the case that intermediate nodes combine information transmitted at different time with time-varying delay. In particular, it has not been clarified when a sink can decode information with any time-varying delay. We showed such sufficient conditions. We also showed the sufficient condition allowing a sink to start decoding of the symbols.

Future tasks include clarifying the maximum achievable rate under the sufficient conditions and constructing the algorithm that composes the network coding satisfying sufficient conditions with the highest rate.

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